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Noise in Ionization Chamber Pulse Amplifiers.

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ABSTRACT.

The general problem of the detection of heavy ionizing particles, in the presence of γ radiation and background noise from the amplifier is considered. It is shown that the maximum possible signal to noise ratio is obtained if the input pulse is shaped to a form $\exp\{-|t-T_D|/\tau\}$. The signal to noise ratio is also calculated for other common pulse shapes.

A practical amplifier circuit is discussed which may be used to detect 100 kv. energy protons in the presence of intense γ radiation.

INTRODUCTION.

THE signal from an ionization chamber is a pulse of current which flows in an external circuit as the movement of ions induces charge on the electrodes. If free electrons are collected, and the counter geometry is properly designed, this time may be as low as $0\cdot 1\,\mu$s. which is less than the usual amplifier time constants. The total charge which passes is proportional to the energy of the initial ionizing particle.

It is desired to amplify this current pulse and to distinguish it from background " noise ". This background noise is due to the superposition of many smaller pulses ; some due to electrons in the counter, others due to the spontaneous thermal agitation of electrons in the electrical circuit (Johnson noise) or the spontaneous movement of electrons in the amplifying valves (shot effect).

The wanted current pulse, like all other current pulses, is transformed into a step pulse of potential on the grid of the first valve, by charging the stray capacity to earth of the ionization chamber and valve. The grid leak of this valve discharges the condenser after a long time, but the grid leak is usually made large in order not to produce appreciable resistor noise, so that this time is sufficiently long to be regarded as infinite.

The amplifier will distort this pulse of potential. Part of this distortion may be deliberate, part caused by defects in the amplifier. It is convenient, however, to consider an ideal, non-distorting, linear amplifier and certain linear pulse-shaping networks which distort this and every other pulse. Then the problem is to decide upon the best way to distort this pulse in order to discriminate against the background noise. It is convenient to "refer" the various sources of background noise to the input as an "equivalent noise charge". This is a fictitious noise charge on the ionization chamber electrodes, which produces the same signal at the amplifier output as the noise.

It will be shown that with a conventional resistance—capacitance pulse shaping network the best signal to noise ratio is obtained when the time constants of "differentiation" and "integration" are equal; that an improvement of 20 per cent is possible by using a shorted delay time for shaping, and that a further improvement of 15 per cent is possible by using a network to shape the pulse to a form $\exp\{-|t-T_D|/\tau\}$. The overall improvement of 35 per cent may be very important.

Analytical Expression for Signal to Noise Ratio.

It will here be assumed that the duration of the current pulse is short compared with the pulse shaping time constants so that the input may be regarded as a function $Q\delta(t)$ of time—where $\delta(t)$ is Dirac's δ function defined by $\delta(t)=0$ for $t\neq 0$ and $\int_{-\infty}^{+\infty}\delta(t)\,dt=1$.

If this assumption is not true (as for example with ion collection in counters) the pulses from the ionization chamber will be attenuated more than the background pulses which is undesirable if a high signal to noise ratio is wanted.

The potential charge on the first valve grid is then $\frac{Q}{c}H(t)$ where $H(t)$ is Heaviside's unit function.

$$H(t)=1 \qquad t>0,$$

$$H(t)=0 \qquad t<0,$$

$$\text{and} \quad \frac{d\{H(t)\}}{dt}=\delta(t).$$

All pulses will be modified by the shaping network. If the input potential pulse is $H(t)$, then the output pulse is a function $f(t)$ of the shaping network. If the input potential pulse is $\delta(t)$, the output pulse will be $d/dt\{f(t)\}=f'(t)$.

The amplifier noise is due to the superposition of the effects of individual electrons in those amplifier elements which are at a low signal level—the input grid leak, the first valve and the first valve anode load. When the ionization chamber is used to count heavy ionizing particles in the presence of intense γ radiation the secondary electron pulses in the chamber also cause a background.

The noise is often expressed in terms of the amplifier bandwidth, here this practice is reversed, and the noise is expressed in terms of the output pulse shape, which is considered to be more fundamental in a pulse amplifier than the bandwidth. This involves a more fundamental consideration of the noise problem.

The valve shot noise may be directly expressed as the superposition of pulses caused by individual electrons in the current stream. Each electron will produce a current pulse as it reaches the electrode; since the transit time ($\sim 10^{-9}$ secs.) is small compared with the amplifier time constants, this may be regarded as a pulse of $e\delta(t)$.

Here can be distinguished two classes of noise. The valve anode has a resistive load R, so that a potential pulse $\text{R}e\delta(t)$ is developed across the load. The superposition of these pulses will be called class I noise. On the other hand, the valve grid has a capacitative load C, and a potential pulse $e\text{H}(t)/\text{C}$ will be developed across the load. The superposition of these other pulses will be called class II. noise.

In practice it is desired to reduce the signal level at which accurate counting is possible. An arbitrary maximum rate of counting of noise peaks is set for each experiment, and the best amplifier conditions must be found in order to reduce the signal level at which this counting rate occurs. A useful concept for this problem is the root mean square noise fluctuation. If the noise current is $\text{I}(t)$ the r.m.s. noise fluctuation σ is given by $\sqrt{\overline{(\text{I}(t)-\overline{\text{I}(t)})^2}}$.

If the rate of arrival of noise pulses is large compared with the frequencies passed by the amplifier, the rate of counting of noise peaks may be expressed in terms of σ, the signal level V, and the amplifier bandwidth. (See equation (20) later). The problem of minimizing the signal level for a constant rate of counting is difficult, but the amplifier bandwidth only enters into the counting rate in a linear term, whereas σ enters in an exponential term. A close approximation, increasingly accurate as V/σ increases, will be to minimize σ. A rough check shows that the pulses should be about 10 per cent wider than given by this criterion, but this would alter the minimum detectable signal by only 1 per cent.

The assumption that the rate of arrival of noise pulses will be greater than the frequencies passed by the amplifier will usually hold for shot noise from both the anode and the grid, and for thermal (resistor) noise. But the γ ray background may not be sufficiently intense for the pulses to "pile up" in this way. In this case the γ rays will be counted as separate pulses of height ke.

Change of pulse shape will not then alter the rate of counting of background at all; so that the γ ray background must be treated as distinct from all other forms of noise. If the pulses are small, their effect will be inappreciable, and if large, they will be similar to unwanted heavy ionizing particles.

An intermediate case will often arise where a certain amount of pile-up occurs which is difficult to treat theoretically, especially since the other forms of noise are added.

If these limitations do not apply, we may proceed by finding the root mean square fluctuation from Campbell's Theorem (Campbell 1909).

Campbell's theorem states that if a noise current $\text{I}(t)$ is caused by the superposition of ν events per second, each causing an effect $f(t)$ then the root mean square fluctuation σ is given by

$$\sigma^2=\overline{(\mathrm{I}(t)-\overline{\mathrm{I}(t)})^2}$$
$$=\nu\int_0^\infty \{f(t)\}^2\,dt. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

The anode current develops a potential difference across a resistance and the pulse $\delta(t)$ later is transformed into a pulse $f'(t)$ by the pulse shaping network. We therefore derive, following Schottky (1918),

$$\sigma_{\mathrm{I}}^2=\mathrm{R}^2e^2\nu\int_0^\infty \{f'(t)\}^2\,dt$$
$$=\mathrm{R}^2\mathrm{I}_a e\int_0^\infty \{f'(t)\}^2\,dt. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

Where σ_{I} is the r.m.s. anode current fluctuation. If we refer this to the ionization chamber input and put in the usual modification for space change limitation (North 1940), we obtain

$$\sigma_{\mathrm{Q}}^2=\frac{0\cdot 12ec^2}{g_m}\,\frac{\mathrm{I}_a}{\mathrm{I}_a+\mathrm{I}_{sg}}\left\{1+\frac{8\mathrm{I}_{sg}}{g_m}\right\}\int_0^\infty \{f'(t)\}^2\,dt\,*$$

(where c is the total input capacity of valve and counter) . (3)

for conventional valves with oxide-coated cathodes and a high amplification factor.

The grid current produces a potential difference across a condenser and the fluctuation becomes

$$\sigma_{\mathrm{Q}}^2=\mathrm{I}_g e\int_0^\infty \{f(t)\}^2\,dt. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

The background secondary electrons in the counter will produce a background

$$\sigma_{\mathrm{Q}}^2=\mathrm{I}_\gamma(ke)\int_0^\infty \{f(t)\}^2\,dt. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

Where $\mathrm{I}\gamma$ is the total ionization current due to γ rays and ke is the average charge collected per electron. The noise in the input grid leak is (following Johnson 1925)

$$\sigma_{\mathrm{Q}}^2=2k\mathrm{T}/\mathrm{R}\int_0^\infty \{f(t)\}^2\,dt. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

which is small compared with the grid current noise if R is large, or more exactly if

$$2k\mathrm{T}/\mathrm{R}\ll(\mathrm{I}_g+k\mathrm{I}_\gamma)e.$$

It can be shown that the anode resistor noise belongs to the same "class" as the anode current noise and is always small in comparison.

* All r.m.s. fluctuations are now expressed in terms of a fluctuation of charge on the first valve grid. This is denoted by use of the suffix σ_{Q}.

Flicker noise cannot be classified in either class I. or class II., but is intermediate. The original process is frequency dependent and σ^2 falls inversely as the first power of the frequency. It will only be appreciable for pulse widths greater than 1 millisecond.

Induced grid noise is of the same class (class I.) as anode noise, but is always negligible in comparison. All the usual forms of noise can therefore be represented in the form

$$\sigma_Q^2=A\int_0^\infty \{f(t)\}^2\,dt+B\int_0^\infty \{f'(t)\}^2\,dt \quad . \quad . \quad . \quad . \quad . \quad (7)$$

where

$$A=(I_g+kI_y)e+2kT/R, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7a)$$

$$B=\frac{0\cdot12eC^2}{g_m}\frac{I_a}{I_a+I_{sg}}\{1+8I_{sg}/g_m\}. \quad . \quad . \quad . \quad . \quad . \quad (7b)$$

For a triode, B reduces to $\dfrac{0\cdot12eC^2}{g_m}$.

Optimum Pulse Shape.

It is often of interest to obtain the maximum possible signal to noise ratio with no restrictions on the shape of the output pulse. This may be simply derived from the expression above by minimizing σ_Q^2 for a constant peak value of $f(t)$. $f(t)$ must clearly be zero at $t=0$ and $t=\infty$ the first from initial conditions, and the second to make

$$\int_0^\infty \{f(t)\}^2\,dt$$

finite.

Moreover it should be a symmetrical pulse; the integral $\int_0^\infty \{f(t)\}^2\,dt$ can be separated into

$$\int_0^{T_D} \{f(t)\}^2\,dt+\int_{T_D}^\infty \{f(t)\}^2\,dt,$$

where $f(t)$ is a maximum at $t=T_D$. If $f(t)$ is symmetrical the two integrals will be equal; if $f(t)$ is not symmetrical and

$$\int_0^{T_D} \{f(t)\}^2\,dt>\int_{T_D}^\infty \{f(t)\}^2\,dt,$$

then σ will be made smaller by altering $f(t)$ between 0 and T to make the pulse symmetrical.

If $f(t)$ is a function of (t/τ) alone where τ is some variable, then the best choice of τ is given when the two integrals which add to produce σ^2 are equal, *i. e.* when

$$A\int_0^\infty \{f(t)\}^2\,dt=B\int_0^\infty \{f'(t)\}^2\,dt. \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

If we can choose the form of the function $f(t)$ then the best function is given by

$$f(t)=\exp\{-|t-\mathrm{T_D}|/\tau\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

and $\tau^2=\mathrm{B/A}$.
This follows by minimizing

$$\int_{\mathrm{T_D}}^{\infty} \mathrm{A}\{f(t)\}^2\,dt+\int_{\mathrm{T_D}}^{\infty} \mathrm{B}\{f'(t)\}^2\,dt=\sigma^2/2$$

with the auxiliary conditions, that at

$$t=\infty \quad f(t)=0,$$

$$t=\mathrm{T_D} \quad f(t)=1,$$

this is normalizing the noise to unit pulse height. Then using the Calculus of Variations we obtain

$$2f(t)=\frac{\mathrm{B}}{\mathrm{A}}\frac{d}{dt}\{2f'(t)\} \quad \text{(Euler's formula)},$$

$$f(t)=\frac{\mathrm{B}}{\mathrm{A}}f''(t),$$

whence $f(t)=\exp\{-(t-\mathrm{T_D})/\tau\}$ where for $t>\mathrm{T_D}$ and $f(t)=\exp\{(t-\mathrm{T_D})/\tau\}$ for $t<\mathrm{T_D}$ by the symmetry property discussed above. There is a discontinuity in $f'(t)$ at $t=\mathrm{T_D}$ but no discontintinuity in $\{f'(t)\}^2$.

Clearly

$$\mathrm{A}\int_0^{\infty}\{f(t)\}^2\,dt=\mathrm{B}\int_0^{\infty}\{f'(t)\}^2\,dt$$

in agreement with formula (8).

From a physical point of view, it is of interest to see why this pulse shape is the best. The sharp pointed peak will not give use to excessive noise, since the extra height adds little to the area under the curve of $\{f(t)\}^2$, whereas it adds considerably to the signal. This pointed peak is the great advantage.

Comparison of Pulse Shapes.

It remains to be seen how much improvement can be obtained in practice by using this pulse shape in preference to other pulse shapes; in order to do this it is necessary to consider the response of commonly used amplifier networks to a pulse $\mathrm{H}(t)$ or $\delta(t)$ to obtain the shape $f(t)$ and $f'(t)$.

The response of the amplifier to a function $\mathrm{H}(t)$ or $\delta(t)$ may be readily found by the method of the Laplace Transformation.

$$\mathrm{F}(p)=\int_0^{\infty} e^{-pt}f(t)\,dt. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (10)$$

The transform of $\delta(t)$ is 1 and of $\mathrm{H}(t)$ is $1/p$.

The system function for the amplifier $G(p)$ is written down by replacing the differential operator d/dt by p. Then the response to a pulse $\delta(t)$ is the inverse transform of $G(p)$ and the response to a pulse $H(t)$ is the inverse transform of $G(p)/p$. The method is discussed by Elmore (1948) and Carslaw and Jaeger (1941). The method is here applied to obtain the signal to noise ratio for a variety of pulse-shaping circuits.

(1) Single differentiating circuit and single integrating circuit.

The system function $G(p)=\tau_2/(1+\tau_1 p)(1+\tau_2 p)$

$$f(t)=(\tau_2/\tau_1)^{1/(1-\tau_2/\tau_1)}\{\exp(t/\tau_2)-\exp(t/\tau_1)\}(\tau_2/\tau_2-\tau_1) \quad . \quad . \quad . \quad (11)$$

when normalized to unity maximum amplitude (Elmore 1948).

σ^2 is then a minimum for $\tau_1=\tau_2=\tau=\sqrt{(B/A)}$ and

$$\sigma^2=\sqrt{(AB)},\ \frac{e^2}{2}=3{\cdot}7\ \sqrt{(AB)}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (12)$$

A similar response is obtained from using a critically damped tuned circuit with $\tau^2=LC$.

(2) A shorted delay line in conjunction with a single integrating circuit gives optimum signal to noise ratio if the delay line time (for twice the line length) is equal to approximately $1{\cdot}2\tau$ where τ is the integrating time constant, and

$$\tau=\sqrt{(B/A)}.$$

Then

$$\sigma^2=2{\cdot}5\ \sqrt{(AB)}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (13)$$

(3) A large number of integrating circuits limit the response of a multi-stage amplifier. After a large number of such circuits the response to a pulse $\delta(t)$ tends to the shape

$$f(t)=\exp\{-(t-T_D)^2/\tau^2\} \quad \text{(Elmore 1948)}, \quad . \quad . \quad . \quad (14)$$

where $T_D=n\tau_1$ and $\tau=\tau_1\sqrt{(n)}$.

Where n is the number of stages. When n is large, this is also the response to a function $H(t)$ if this has been differentiated with a time constant small compared with τ. Pulse shaping with a shorted delay line and a large number of integrating circuits will also give approximately the same shape. Then

$$\sigma^2=2{\cdot}5\sqrt{AB} \text{ with } \tau=\sqrt{A/B}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (15)$$

The optimum case with

$$f(t)=\exp\{-\,|\,t-T_D\,|\,/\tau\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (15a)$$

with $\tau=\sqrt{A/B}$ gives

$$\sigma^2=2\sqrt{AB}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (16)$$

These figures differ from those derived by Elmore (1948). Elmore defines a pulse width

$$W^2 = \frac{2\pi \int_0^\infty (t-t_0)^2 f(t)\, dt}{\int_0^\infty f(t)\, dt}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (17)$$

where

$$t_0 = \frac{\int_0^\infty t f(t)\, dt}{\int_0^\infty f(t)\, dt} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (17a)$$

and derives the minimum noise—neglecting the grid noise (*i. e.* putting A=0) for a constant value of W. It is of interest to compute W for our various pulse shaping networks when the parameter t_0 has been chosen to make σ^2 a minimum.

For the optimum case $W^2 = 4\pi \frac{A}{B}$.

For the Gaussian case $W^2 = \pi \frac{A}{B}$.

For the RC–RC case $W^2 = 12\pi \frac{A}{B}$.

For the RC–Delay line case $W^2 = 4\pi \frac{A}{B}$.

Thus the Gaussian case has the advantage of giving a smaller pulse width—according to Elmore's definition. This is in fact because there is less of a " tail " to the pulse.

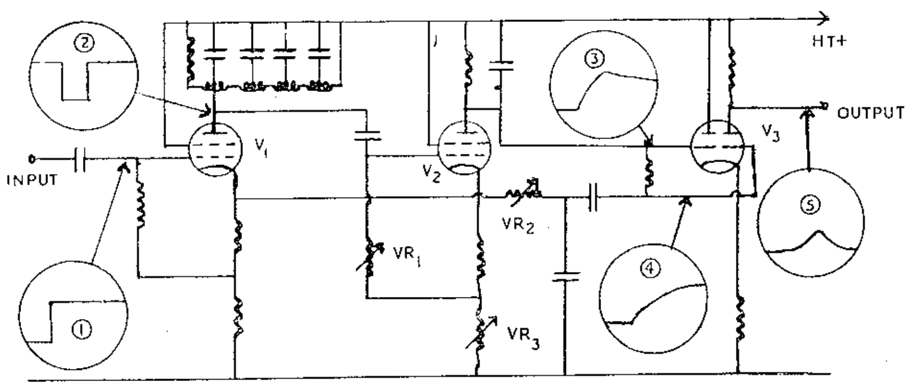
For most purposes it will not be necessary to produce a circuit to shape the pulse to the optimum. The improvement over a Gaussian or RC–Delay line case is only 15 per cent. For some experiments, however, this extra 15 per cent may make all the difference. This arises when low energy protons are to be counted in the presence of intense γ radiation. (Wilson, Collie and Halban 1949). A circuit has been devised (fig. (1)) which will shape the pulses to something which closely approaches the optimum shape. It is necessary, for such a circuit, that only *linear* networks be used ; otherwise, although a pulse H(t) may be shaped to a pulse, $f(t)$ a pulse 2H(t) will not be shaped to a pulse $2f(t)$.

Briefly the operation of the circuit is as follows. The input pulse (1) is split into two parts. One half is converted to a square pulse (2) and then into a nearly linear voltage rise (3). The other half is converted into an exponential rise (4). The first shape is controlled by the delay length and VR1, the second shape by VR2. The relative amplitudes are controlled by the feedback resistor VR3, so that the initial slopes of both waveforms (3) and (4) are the same. The double triode V_3 is connected so that the difference between the two waveforms is taken as the output. The pulse shape after the peak is exactly the optimum shape, but that before the peak rises too quickly (theoretically it should begin to rise an infinite sime before the peak). With suitable choice of constants

$$\sigma^2 = 2\cdot1\sqrt{AB}. \quad \ldots\ldots\ldots \quad (18)$$

This computation made above of the optimum pulse shaping network assumes that the valve anode and grid noise are independent and constant. If the valve operating conditions are altered, the grid current noises can be reduced, at the expense of the anode current noise, by reducing the HT voltage (and hence the space current) and also by slightly reducing the filament voltage. The minimum noise is given when AB is a minimum (provided that the amplifier time constants are readjusted to the optimum values). If only valve noise and γ background are important this means $(kI_\gamma + I_g)/g_m$ must be a minimum. This is a slight extension of a formula of Gillespie (1948). This assumes that the anode current noise is completely space charge limited, and therefore only holds for a triode.

Fig. 1.



Pulse shaping circuit.

If two different valves are to be compared, then the input capacity alters. A "figure of merit" for a valve in any given application is therefore

$$g_m/C^2(I_g + kI_\gamma). \quad \ldots\ldots\ldots \quad (19)$$

Where C is the total input capacity of valve and counter (but *not* the feedback capacity due to Miller effect).

Counting of Noise Peaks.

Before it is possible to compare these results with practice it is necessary to consider the rate of counting of noise peaks.

The number of times that the noise waveform passes through a voltage V with positive slope is

$$[\exp\{-V^2/2\overline{V_0^2}\}]\{\tfrac{1}{2} \text{ expected number of zeros of the noise waveform}\}$$

(Rice 1944, 1945, Middleton 1948). . . . . . (20)

This is clearly the number of times the discriminator triggers.

The expected number of zeros per second is

$$2\left[\frac{\int_0^\infty f^2 W(f)\,df}{\int_0^\infty W(f)\,df}\right]. \qquad \qquad (21)$$

Where $W(f)\,df$ is the noise power at frequencies between f and $f+df$.

For the cases considered this approximates to $0{\cdot}6/\tau$ where $\tau=\sqrt{B/A}$. (The case of an integrating R–C circuit as the upper frequency cut off, gives a divergent integral; this is discussed by Rice but will not arise in physically realizable circuits.)

For most of our cases $\tau=10^{-6}$ secs.

Then the number of zeros is $0{\cdot}6\times10^6$/second.

Then there will be less than 1 count per minute on noise peaks if $V/V_0>6$, *i. e.* if the signal is greater than 6 times the r.m.s. noise.

Practical Amplifier Circuit.

The above analysis has been carried out in order to design a counter-amplifier system to count low energy protons in the presence of intens γ radiation for work on photodisintegration of deuterium (Wilson, Collie and Halban, 1949). The valve chosen for first valve is an American 6AK5 connected as a triode by strapping screen and anode. This reduces the anode current shot noise. The total input capacity of valve and counter is 13 pF. The grid leak is made up only of the valve and insulator leakages and is so high that no noise arises.

The 6AK5 valves used were well aged, and then selected; the grid current was then smaller than any other type tested; the Mullard EF37, although it has a low grid current, gives more anode current noise, and the EC91, or EF91, although they give little anode current noise have a higher input capacity and more grid current noise.

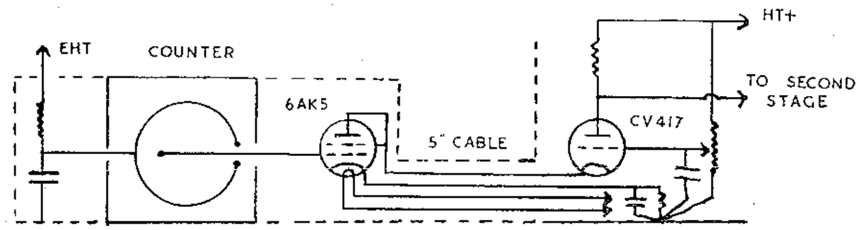
Johnson and Johnson (1936) and Parsegian (1946) recommend pentodes for first valve to avoid the " Miller effect " feedback. This feedback *does not* affect the signal to noise ratio, but it affects the gain and may result in accentuation of second valve noise. This can be obviated by a compensating positive feedback from the second valve anode to the first valve grid (Schultz 1946). We have, however, inserted an extra valve in the " cascode " connection (Hunt and Hickman 1939, Wallman, MacNee and Gadsden 1948). The anode of the first valve feeds directly to the cathode of the second which represents a sufficiently low impedance (100 Ω) to prevent appreciable amplification and feedback by the first valve. Yet the second valve noise is reduced by the high impedance of the first valve in the cathode. This circuit also enables the first valve to be placed away from the subsequent valves—separated by a yard or so of cable (fig. 2).

After amplification by a feedback ring a 100 Ω line is taken to the main amplifier where the pulse shaping networks are situated. A shorted delay line and resistance capacitance integration are normally used. It is possible, with selected valves, to make grid and anode current noise equal with $\tau=0{\cdot}5\times10^{-6}$ secs. with $g_m=4$ mA/V. Under these conditions, the equivalent number of ion pairs for the noise$=\sigma/c=300$.

Is is therefore possible to count pulses of ionization of 2000 ions pairs above background, or 70 keV.

When a γ ray source is brought near the high pressure counter used, the value of τ is reduced, to $0{\cdot}3\times10^{-6}$ secs. (below which loss of pulse height might occur) and it is only possible to count down to about 90 keV. These values of the background, and the rate of counting of noise pulses (equation (20)) are fully borne out experimentally, confirming that there are no other forms of background than the statistical ones considered here.

Fig. 2.



Amplifier input circuit.

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References.

Campbell, N., 1909, *Proc. Camb. Phil. Soc.*, **15**, 117.
Carslaw, H. S., and Jaeger, J. C., 1941, *Operational Methods in Applied Mathematics.* O.U.P.
Elmore, W. C., 1948, *J. App. Phys.*, **18**, 55 ; *Ibid.*, *Nucleonics*, **2**, 23.
Gillespie, A. B., 1948, *A.E.R.E. Report.*
Hunt, F. V. and Hickman, R. W., 1939, *Rev. Sci. Inst.*, **10**, 6.
Johnson, J. B., 1925, *Phys. Rev.*, **26**, 71.
Johnson, E. A., and Johnson, A. G., 1936, *Phys. Rev.*, **50**, 175.
Middleton, D. O., 1948, *J. App. Phys.*, **19**, 817.
North, D. O., 1940, *R.C.A. Rev.*, **5**, 244.
Parsegian, V. L., 1946, *Rev. Sci. Inst.*, **17**, 39.
Rice, S. O., 1944, *Bell System Tech. Jnl.*, **23**, 282 ; 1945, *Ibid.*, **24**, 146.
Schottky, W. 1918, *Ann. Phys. Lpz.*, **57**, 541.
Schultz, H. L., 1946, *Phys. Rev.*, **69**, 689.
Wallman, H., MacNee, A. B., and Gadsden, C. B., 1948, *Proc. Inst. Rad. Eng.*, **36**, 700.
Wilson, R., Collie, C. H., and Halban, H. H., 1949, *Nature*, **163**, 245.